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COMMENT

One-dimensional random potentials allowing for extended states

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Abstract. We present a class of random potentials that allow for extended states at selected energies. These potentials can be obtained by starting from an underlying non-disordered potential and inserting in it in a random way particular potential profiles defined on a finite length. By making a suitable choice of the potentials to be inserted the energies of the extended states can be fixed arbitrarily.

A great deal of work has shown that for one-dimensional random potentials all electronic states that are solutions of the Schrödinger equation are, in general, localised [1]. However, Denbigh and Rivier [2] have found a particular class of random potentials that allow for extended states. It is, therefore, of interest to identify the potentials with this property in order to understand under which conditions the randomness is not sufficient to localise all the solutions of the Schrödinger equation. In this Comment we present a more general class of such one-dimensonal potentials and show that the potentials given by Denbigh and Rivier belong to it as particular cases.

The potentials $V(x)$ of this class can be described by giving an iterative procedure to construct them. These potentials are built by starting from a non-random underlying potential $U(x)$ defined on the interval $(-\infty, \infty)$. We assume that $U(x)$ has extended states. The potential $V(x)$ is obtained by making successive random insertions in $U(x)$ of a potential profile $W(x)$, which is defined on a finite length L. This is done in the following way. In the first step, we randomly determine a position x_1 and construct the potential $V^1(x)$:

$$V^1(x) = \begin{cases} U(x) & \text{for } x < L \\ W(x) & \text{for } x_1 \leqslant x \leqslant x_1 + L \\ U(x - L) & \text{for } x > x_1 + L. \end{cases} \tag{1}$$

In the second step we determine x_2 and insert, in the same way, $W(x)$ in $V^1(x)$—and so on until we reach a finite density of insertions on the whole real axis.

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In general, this procedure will localise all the extended states of the original potential $U(x)$. However, we will show that under particular conditions on $W(x)$ this randomness is insufficient to localise all these extended states. To determine such conditions we study the problem by the transfer matrix method. We define the transfer matrix $\mathbf{M}(x, x'; E)$ for transfer from x to x' as follows:

$$\begin{pmatrix} \psi(x') \\ \psi'(x') \end{pmatrix} = \mathbf{M}(x, x'; E) \begin{pmatrix} \psi(x) \\ \psi'(x) \end{pmatrix} \tag{2}$$

where ψ is the wavefunction and ψ' its derivative. The matrix $\mathbf{M}(x, x'; E)$ depends on the energy E and on the potential on the interval $[x, x']$.

If we define an arbitrary set of points $\{x_n\}$ that satisfies $x_{n+1} > x_n$ and $\lim_{n\to\infty} x_n = \infty$, the localisation length ξ, which is the reciprocal of the Lyapunov exponent of the product of the matrices $\mathbf{M}$, can be expressed as [3]

$$\xi^{-1} = \lim_{n\to\infty} |x_n - x_1|^{-1} \log \left| \prod_{i=1}^{n} \mathbf{M}_n(x_n, x_{n+1}; E) \begin{pmatrix} \psi(x_1) \\ \psi'(x_1) \end{pmatrix} \right|. \tag{3}$$

The Oseledec theorem [4] ensures that ξ exists and does not depend on the initial conditions $(\psi(x_1), \psi'(x_1))$ nor on x_1. The choice of the set $\{x_n\}$ is free because the continuity of the wavefunction and of its derivative ensure that, for $x < x' < x''$,

$$\mathbf{M}(x, x'; E)\mathbf{M}(x', x''; E) = \mathbf{M}(x, x''; E). \tag{4}$$

This property allows us to choose the extrema of the insertion potentials, $W(x)$, as $\{x_n\}$.

If, for a particular energy E^*, the transfer matrix $\mathbf{M}_W(0, L; E^*)$ for transfer across the potential W is equal (apart from an overall phase) to the identity matrix

$$\mathbf{M}_W(0, L, E^*) = e^{i\theta}\mathbf{I} \tag{5}$$

its contribution to the matrix product in (3) is trivial. Thus the localisation length ξ_V in the potential $V(x)$ at the energy E^* is related to ξ_U in $U(x)$ by

$$\xi_V(E^*) = (1/\rho_U)\xi_U(E^*) \tag{6}$$

where ρ_U is the fraction of length occupied by $U(x)$ in $V(x)$. From (6) it follows that if there exists an extended state in $U(x)$ at E^*, there will be a corresponding extended state in $V(x)$ at the same energy.

The same conclusion is reached if we build the wavefunction with the potential $V(x)$ at the energy E^* starting from the wavefunction of $U(x)$ at the same energy. Condition (5) on $\mathbf{M}_W(0, L, E^*)$ is equivalent to stating that the Schrödinger equation for $W(x)$ on $[0, L]$ with arbitrary periodic boundary conditions must have solutions at energy E^*. The wavefunction of $V(x)$ at E^* is therefore everywhere equal to that of $U(x)$, except in the insertions where it is matched with the appropriate solution of $W(x)$.

We stress that to obtain extended states in $V(x)$, the potentials $W(x)$, which are inserted in $U(x)$, do not need to be all equal. In fact the only condition we require is that they satisfy equation (5) at the same energy E^*. In particular, the $W(x)$ can even be chosen randomly from a set of potentials, as we will show. We remark that the matrix $\mathbf{M}_W$ does not change if a constant potential is added to $W(x)$. However, the resonant energy E^* will shift correspondingly. Therefore by adding a suitable constant potential to the insertion potentials W we can fix the resonant energy at a selected energy level of an extended state of $U(x)$.

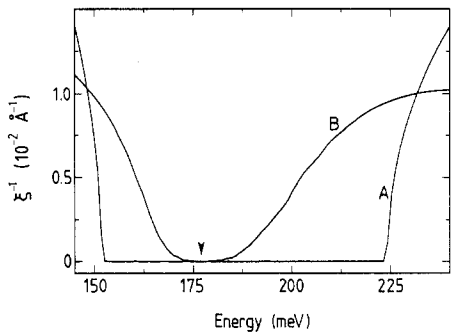

Figure 1. The reciprocal of the localisation length as a function of energy for the model described in the text. Curve A refers to the periodic Kronig–Penney potential; curve B to the random system obtained by insertions of a flat zero potential. The arrow indicates the resonant energy E^*.

As an example, we take as the underlying potential $U(x)$ a periodic Kronig–Penney potential with barriers of finite height U_b. All solutions of the Schrödinger equation are extended and the allowed energy levels can be grouped into bands. As the insertion potential $W(x)$, we consider a flat zero potential on a length L. The resonance condition (5) is satisfied for energies

$$E_n = \pi^2\hbar^2 n^2/2mL^2 \tag{7}$$

where m is the particle mass and n is an integer. We choose the energy E^* in an energy band of the periodic potential. Then, in order to obtain an extended state in $V(x)$ at this energy, we take the length L such that $E^* = E_n$ for an integer n. All the extended states of the periodic potential will localise, except those at energies E_n, and in particular the one at E^*. In this way, the random insertions act as a filter for the selected energy. We notice that if $E^* < U_b$ we obtain perfect transmission by coherent tunnelling in a random potential. For this case we present in figure 1 numerical results that show how the localisation length for states of energies close to the selected energy E^* is modified by the random insertions of potentials W. The results reported in the figure refer to GaAs/$Ga_{1-x}Al_xAs$ heterostructures. The motion of conduction electrons in these materials is subject to a one-dimensional potential profile, which can be produced artificially [5]. We considered as $U(x)$ the potential of a periodic 21.4 Å/21.4 Å GaAs/$Ga_{0.6}Al_{0.4}As$ superlattice, and as $V(x)$ the potential of the random heterostructure obtained by increasing the length of each GaAs layer (potential well) by $n \times 56.6$ Å where n takes the values 0, 1, 2, 3 with equal probability. In the random system, the resonance occurs at $E^* = 177$ meV. From figure 1, we see that all the states except the one at E^* are localised by the randomness.

We present, as another example, a case where not only are the insertions random, but also the insertion potentials $W(x)$ are chosen randomly from a set of potentials. We consider an underlying flat zero potential and insert, randomly, square wells or barriers. The height of the inserted potential W_L (positive for barriers and negative for wells) may vary in a given range $[W_{min}, W_{max}]$, and the length L is related to W_L by

$$E^* = \hbar^2\pi n^2/2mL^2 + W_L \tag{8}$$

where $E^* > W_{max}$ is a constant and n is an arbitrary integer. This system presents only one extended state, at the selected energy E^*.

Denbigh and Rivier [2] considered a Kronig–Penney potential where the barriers, all of the same height and width, are located at random distances from each other. They showed that this random potential allows for an infinite discrete spectrum of extended states with energies above the barriers. This potential belongs to the class of potentials

described above. In fact, it can be obtained from the last example by taking $W_{\min} = W_{\max} = W > 0$ and fixed L. Since W and L are fixed, all the E^* obtained from (8) for different n correspond to extended states.

Denbigh and Rivier generalised this case by considering, instead of square barriers, other potentials that allow perfect transmission at some energy E^*. In this case $\mathbf{M}_W(E^*)$ does not necessarily satisfy condition (5). However, if the underlying potential is flat and if all the insertions of $W(x)$ are separated from each other, weaker conditions on the matrix $\mathbf{M}_W$ can be found. In fact if, for an energy E^*, the matrix $\mathbf{M}_W(0, L; E^*)$ corresponds to a transfer matrix for transfer across a flat potential of arbitrary length (perfect transmission), an extended state at E^* will be found in $V(x)$. This can be seen from equation (3), since now, at this energy, all matrices represent transfer across a flat potential.

In conclusion, we have described a class of one-dimensional random potentials that allow for extended states. We have shown the existence of random potentials that are able to localise all states but one. By constructing these potentials appropriately, we can fix the energy of this state arbitrarily.

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